INTENSIFICATION OF MASS TRANSFER IN A CURRENT-CARRYING FLUID AT HIGH PÉCLET NUMBERS

A. D. Polyanin and P. A. Pryadkin UDC 532.72

Two axisymmetric problems relating to mass transfer in a system consisting of a solid body and an electrically conducting fluid are examined.

In the diffusion boundary layer approximation analytical solutions are obtained for two steady-state problems of convective diffusion to the surface of nonconducting solid bodies in a flow of viscous electrically conducting fluid. In [1-8] the same approximation was used to investigate diffusion for different modes of flow of a viscous incompressible fluid over a particle. In [7-8] the fluid was assumed to be electrically conducting and the effect of the electromagnetic field was taken into account.

1. We consider steady convective diffusion to the spherical surface of a solid body in an axisymmetric laminar flow of viscous incompressible fluid. We assume that the Péclet number is high (diffusive transfer of matter over the surface of the solid can then be neglected in comparison with normal transfer); the surface of the solid body completely absorbs the dissolved substance in the liquid, and in the flow core (outside the diffusion boundary layer) the concentration is constant.

The aim of the present work was to calculate the total diffusive flows to a spherical surface in the two special cases dealt with below.

With a prescribed stream function we can use the results of [3] (obtained in the diffusion boundary layer approximation), which for the dimensionless total flow to the part of the spherical surface enclosed between angles θ^- and θ^+, have the form

$$I = \pi 6^{1/3}\Gamma^{-1}\left(\frac{4}{3}\right) A^{2/3}(\theta^+,\ \theta^-)\,\mathrm{Pe}^{1/3},$$
$$A(\alpha,\ \beta) = \left|\int_\alpha^\beta \sin\tau\,\left|f(\tau)\right|^{1/2} d\tau\right|,\quad f(\theta) = \frac{1}{2}\left[\frac{\partial^2\psi}{\partial r^2}\right]_{r=1} \tag{1}$$

The characteristic scales here are the radius of the spherical surface, the characteristic flow velocity, and the concentration outside the diffusion boundary layer (in the flow core). Adjacent points of inflow and outflow correspond to angles θ^- and θ^+.

The inflow (outflow) point is the critical point of the body surface, in whose vicinity the normal velocity component of the fluid is directed toward (away from) the surface. Angles θ^- and θ^+ are given by the following relations [3]:

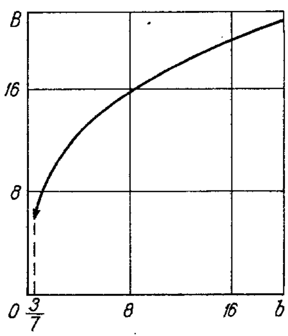

Fig. 1. Relation B(b).

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$$f(\theta^-) = 0,\ H(\theta^-) < 0;\ f(\theta^+) = 0,\ H(\theta^+) > 0;\ \sin\theta\, H(\theta) \equiv df/d\theta. \quad (2)$$

2. We investigate steady-state convective diffusion of matter to the surface of a solid nonconducting sphere in a flow of viscous electrically conducting fluid on the assumption that far from the sphere the liquid velocity U and electric current density j are constant and have the same direction.

The flow field of this problem for low Reynolds numbers was obtained in [9] by the method of matched asymptotic expansions:

$$\psi = (1/4)(r-1)^2 \sin^2\theta \{[1 + (3+\varkappa)\,\mathrm{Re}/8\,](2 + r^{-1}) -$$
$$- (3/8)\,\mathrm{Re}\cos\theta\,[2 + r^{-1} + r^{-2} + (\varkappa/3)(2 + 4r^{-1} + r^{-2})]\},\ \varkappa = O(1),$$
$$f(\theta) = \frac{1}{32}\mathrm{Re}\,(12 + 7\varkappa)\sin^2\theta\,(b - \cos\theta), \quad (3)$$
$$b = \left(\frac{8}{\mathrm{Re}} + 3 + \varkappa\right)\left(4 + \frac{7}{3}\varkappa\right)^{-1};\ \frac{3}{7} < b < \infty,$$

where the function $f(\theta)$ is defined as in (1).

When $3/7 < b < 1$ a region of closed circulation is formed in the rear of the sphere, but when $b \geq 1$ this region is absent. Using relations (2), we obtain the following values for the angles θ^- and θ^+:

$$\theta^- = \pi,\ \theta^+ = \begin{cases} \arccos b, & b < 1, \\ 0, & b \geqslant 1. \end{cases}$$

Using expression (1), we obtain the total diffusive flow to the sphere

$$I = \left[\frac{1}{8}\mathrm{Re}\left(4 + \frac{7}{3}\varkappa\right)\right]^{1/3} B(b)\,\mathrm{Pe}^{1/3},$$

$$B(b) = \begin{cases} 1.91\,[2(b^2+3)E(k_1) - (3-b)(1-b)F(k_1)]^{2/3}, & 3/7 < b < 1, \\ 2.41\,(1+b)^{1/3}[(b^2+3)E(k_2) - b(b-1)F(k_2)]^{2/3}, & b \geqslant 1, \end{cases} \quad (4)$$
$$B(3/7) = 5.41,\ B(b\to\infty) \to 3.25\,b^{2/3},$$
$$k_1^2 = (1+b)/2,\ k_2^2 = 2/(1+b),$$

where $F(\lambda)$ and $E(\lambda)$ are complete elliptic integrals of the first and second kind, respectively.

The relation B(b) for $3/7 < b \leq 20$ is shown in Fig. 1. In the calculation of the total diffusive flow to the sphere, the region of closed circulation was ignored.

It is apparent that with increase in current density the total diffusive flow to the particle increases, e.g., when Re = 0.5 an increase in $\varkappa$ from 0 to 10 leads to an increase in the flow by 13%.

3. We consider diffusion of matter to the inner surface of a hemispherical ladle completely filled with viscous electrically conducting fluid. We assume that the ladle is not electrically conducting, and in the center of the flat free surface there is a point source of current of constant strength J_0, causing a flow within the ladle (Fig. 2).

In the Stokes approximation the stream function of such a flow is obtained in the form of a series [10]

$$\psi = (1-\mu^2)\,r\,(1-r^2)^2 \sum_{n=1}^{\infty} a_{2n}(1 + 2r^2 + 3r^4 + \ldots + nr^{2n-2})\frac{d}{d\mu}P_{2n}(\mu), \quad (5)$$
$$a_{2n} = -(4n+1)P_{2n}(0)\,[4n^2(n+1)(2n-1)(2n+1)^2]^{-1},\ \mu = \cos\theta,$$

where the stream function is dedimensionalized with respect to the characteristic velocity.

In the course of solution of the ladle surface the concentration will vary continuously with time and, hence, for the fluid contained in the ladle diffusion will be unsteady. The characteristic time of variation of the

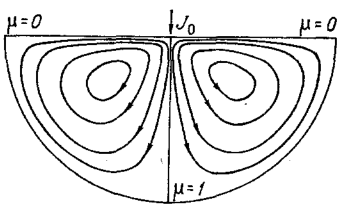

Fig. 2. Streamlines in ladle.

concentration in the flow core (outside the diffusion boundary layer) will be on the order of $V/(4\pi a D \mathrm{Pe}^{1/3})$ and the time to establish diffusion in the boundary layer is $t_D \sim a^2 \mathrm{Pe}^{-2/3} D^{-1}$. Hence, when $\mathrm{Pe}^{-1/3} \ll 1$ diffusion in the boundary layer can be regarded as quasisteady, but the variation of the concentration with time is parametric.

It should be noted that inside the ladle the flow proceeds along closed streamlines and, hence, the condition of inflow in the vicinity of the critical point $\theta^- = 0$ is given by the concentration carried by the flow from the interior of the solution along the streamlines given by the relations $\psi < 0$ $(\mathrm{Pe}^{-2/3})$. It was shown in [11] that equalization of the concentration of substance arriving from the diffusion boundary layer to its value in the flow core occurs at a dimensionless distance on the order of $\mathrm{Pe}^{-1/9} \ll 1$ from the body surface. Hence, when $\mathrm{Pe}^{-1/9} \ll 1$ the usual condition of nondepletion of the solution is satisfied in the vicinity of the inflow point.

We note here that the solidity of the dissolving surface plays a significant role in equalization of the concentration. For a flow with closed streamlines occurring near a dissolving liquid surface there is no equalization, and transfer of the diffusing substance is more complex, e.g., [12-14].

For function $f(\theta)$ we obtain

$$f(\theta) = 2(1-\mu^2) \sum_{n=1}^{\infty} a_{2n} n(n+1) \frac{d}{d\mu} P_{2n}(\mu). \tag{6}$$

Using expressions (2) we have $\theta^- = 0$, $\theta^+ = \pi/2$. For the dimensionless total diffusive flow onto the inner surface of the ladle

$$I = 2.96\,\mathrm{Pe}^{1/3} = 2.96\, J_0^{2/3} (\pi\rho\nu D)^{-1/3}. \tag{7}$$

It is apparent from (7) that the total diffusive flow of substance increases in proportion to $J_0^{2/3}$, i.e., with increase in the strength of the current source the flow of diffusing substance can be greatly increased.

We now determine the law of variation of the concentration in the flow core with time. In unit time the substance dissolved in the ladle (in the flow core) changes by an amount $V\,|dc_0/dt|$, equal to the total diffusive flow to the ladle surface aDc_0I.

This gives the following equation for the concentration

$$dc_0/d\tau = -1.41\,\mathrm{Pe}^{-2/3} c_0, \quad c_0(\tau = 0) = 1, \tag{8}$$

whose solution has the form

$$c_0(\tau) = \exp(-1.41\,\mathrm{Pe}^{-2/3}\tau). \tag{9}$$

Expression (9) shows that the concentration in the flow core slowly decreases with time from unity to zero.

NOTATION

a, radius of body; U, characteristic flow velocity; ρ, density of fluid; ν, viscosity; ψ, dimensionless stream function; c_0, concentration in flow core; c, dimensionless concentration; D, diffusion coefficient; j, electric current density; J_0, current strength; μ_e, magnetic permeability of fluid; $\mathrm{Re} = aU\nu^{-1}$, Reynolds number; $\mathrm{Pe} = aUD^{-1}$, Péclet number; $\varkappa = \mu_e j^2 a^2 \rho^{-1} U^{-2}$, dimensionless parameter; $\Gamma(s)$, gamma function; $P_{2n}(\mu)$, Legendre polynomial of degree 2n; t, time; $\tau = Ut/a$, dimensionless time; $V = (2/3)\pi a^3$, volume of ladle.

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ROLE OF RHEOLOGY IN THE EXTENSION OF POLYMER MELTS BY A CONSTANT FORCE

A. N. Prokunin and N. G. Proskurnina

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The uniform extension of an elastic liquid by a constant force is experimentally investigated, and the experiment is compared with theory.

In [1] a system of equations with four rheological constants was written to describe any noninertial uniform extension.* These equations were as follows

$$\frac{1}{\lambda}\,\frac{d\lambda}{d\tau}+\frac{(\lambda+1)(\lambda^3-1)}{6\lambda^2}\exp(-L)=\Gamma(\tau),$$

$$L=\frac{\beta}{2\lambda^2}(\lambda-1)^2(\lambda^2+4\lambda+1),\quad(\tau=t/\theta;\ \Gamma=\varkappa\theta),\tag{1}$$

$$\sigma=\frac{\sigma'\theta}{\eta}=(1-s)(\lambda^2-\lambda^{-1})+3s\Gamma\exp(L).$$

These equations were derived using the classical potential of the grid theory of high elasticity.

In the extension of a sample by a constant force F, one end is rigidly fixed, and the other moves under the action of F (a diagram is shown in Fig. 1). In this case, the dimensionless stress is

$$\sigma=\sigma_0\,\frac{p_0}{p},\quad\sigma_0=\frac{\theta F}{\eta p_0}.\tag{2}$$

The expression for the deformation rate under tension is $\Gamma=(1/l)(dl/d\tau)$ [3]. Using the incompressibility conditions for the liquid, $p_0l_0=pl$, it may be written in the form

$$\Gamma=-\frac{1}{p}\cdot\frac{dp}{d\tau}.\tag{3}$$

Differentiating Eq. (2) with respect to τ and using Eq. (3), the following result is obtained

*Surface tension was neglected.

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